Giant density fluctuations in locally hyperuniform states

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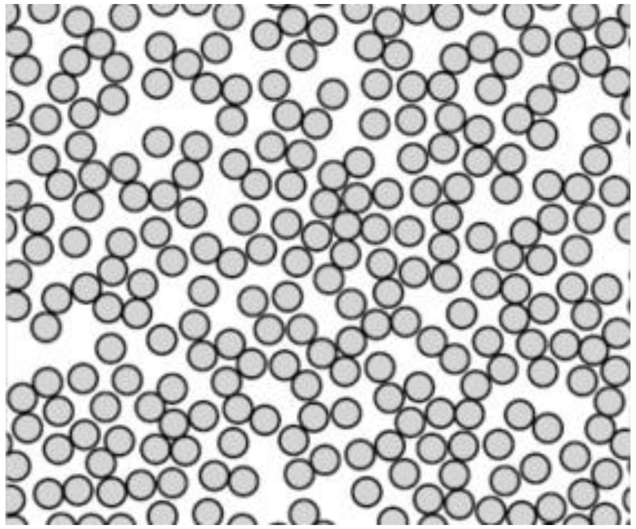
Normal density fluctuations



- Large system of size L with a fixed number N of particles
- Subsystem of size $\ell \ll L$ with a fluctuating number n
- Varying ℓ, one has: $\langle(\Delta n)^2\rangle_\ell \sim \langle n \rangle_\ell$ (normal fluctuations)

Variance proportional to mean number of particles in the box

—> Can be tested experimentally or numerically

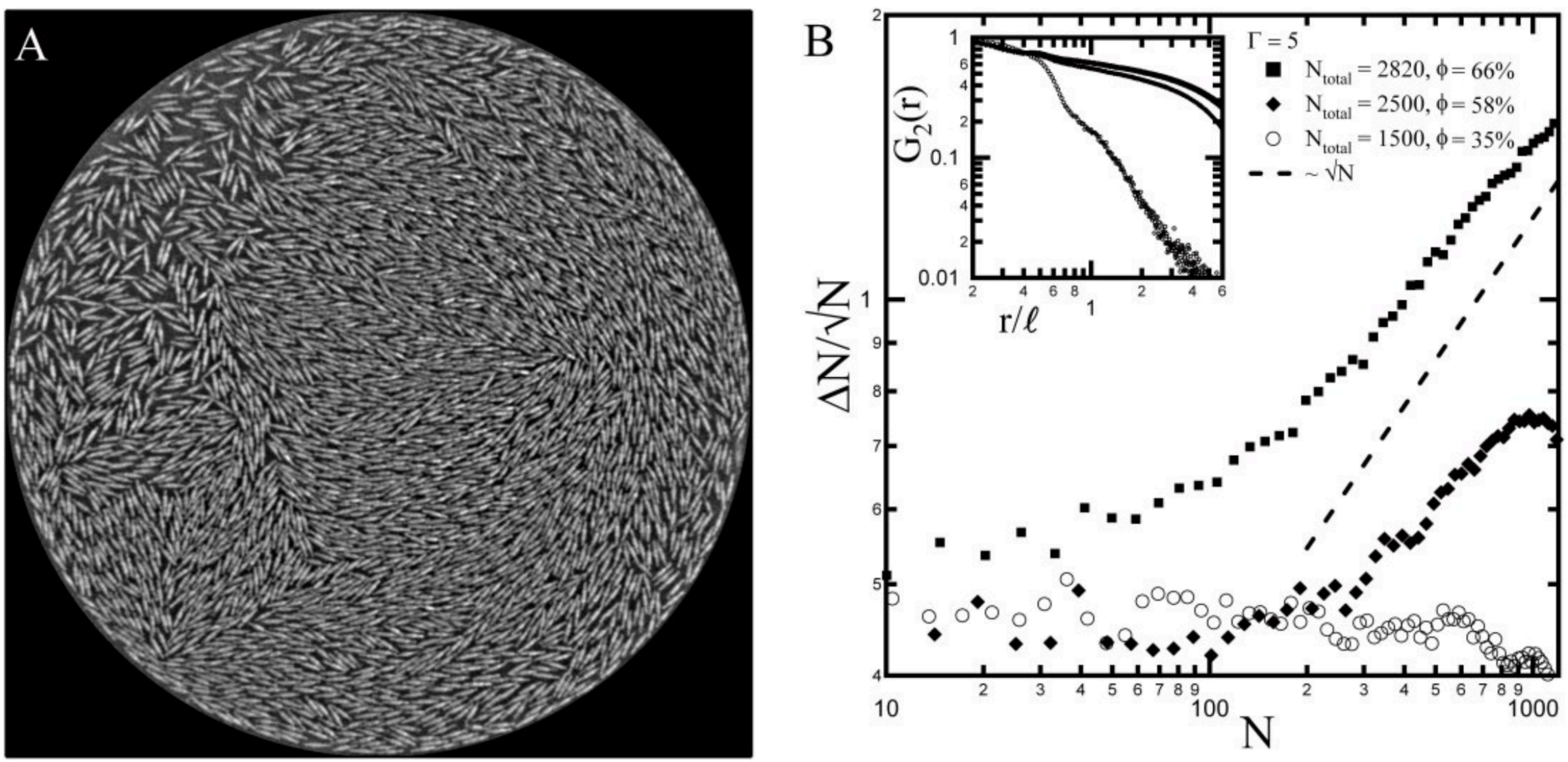
- Alternative characterization: structure factor

$$S(\boldsymbol{q}) = N^{-1}\langle \textstyle\sum_{i,j} e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)}\rangle$$ for N particles at position $\mathbf{r}_i$

Link with fluctuations of n: $S_{\mathrm{iso}}(q) \sim \dfrac{\langle \Delta n^2 \rangle_\ell}{\langle n \rangle_\ell}$ with $q \sim \dfrac{2\pi}{\ell}$

Normal fluctuations: $S_{\mathrm{iso}}(q) \to S_0 > 0$ when $q \to 0$

Giant density fluctuations



Anomalously large density fluctuations

Nematic order in a vibrated layer of elongated grains

Driven stationary state

+

Orientational order

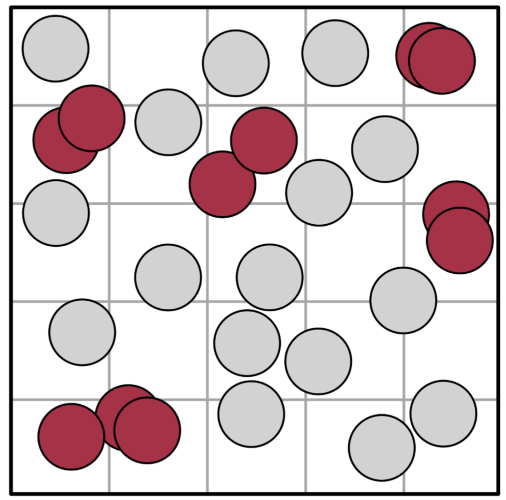
=

Giant density fluctuations

Narayan, Ramaswamy, Menon, Science (2007)

Anomalously low density fluctuations

Motivation:
Suspensions under oscillatory shear
Pine *et al.*, Nature (2005)



- Random Organization Model:
  Only overlapping particles can move
- Transition between passive and active state as a function of packing fraction ϕ

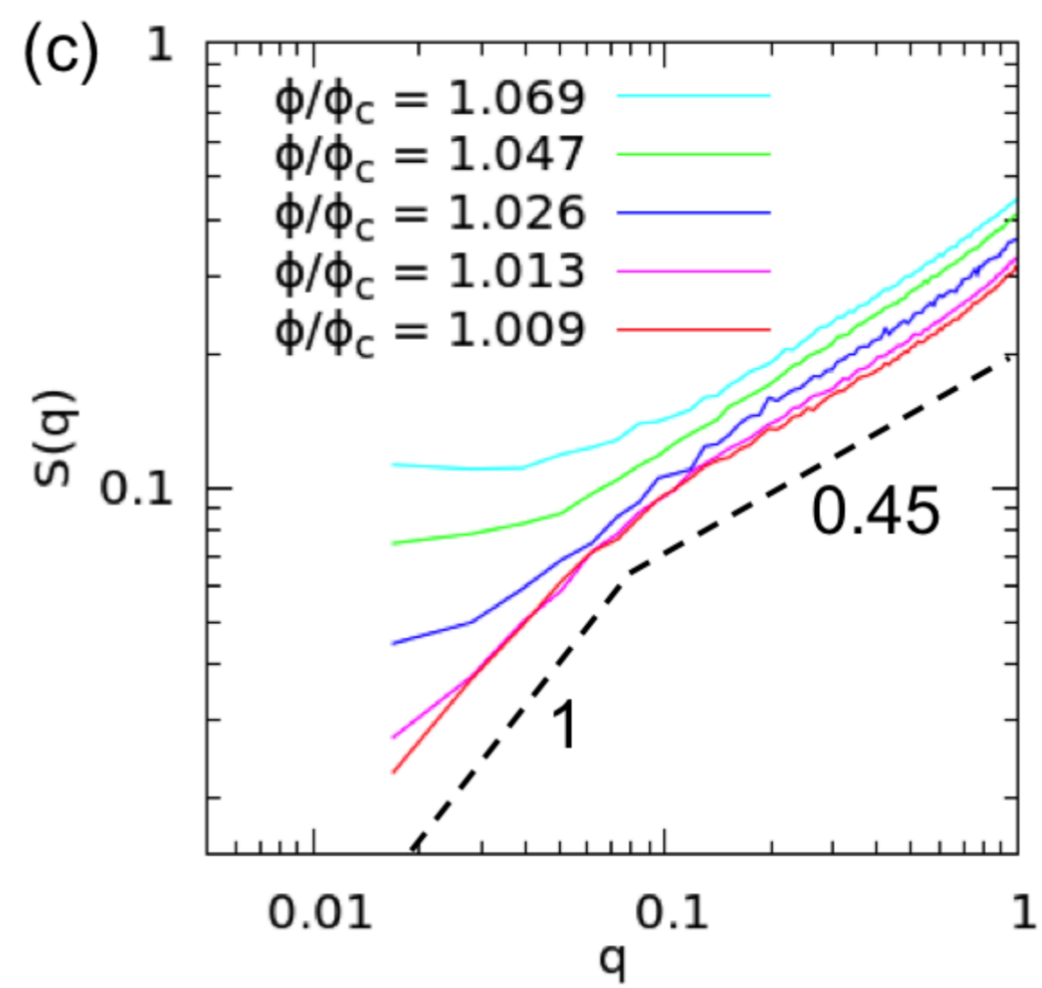
Tjhung, Berthier, PRL (2015)

At the critical point ϕ_c:
Structure factor $S(q) \to 0$

Anomalously low fluctuations

= Hyperuniformity

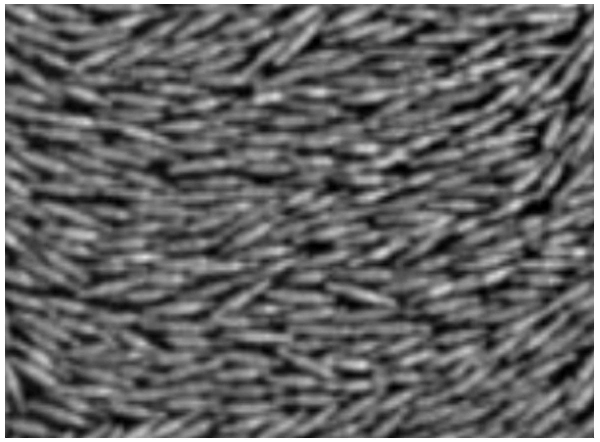
Spatial organization of particles
Intermediate between liquid and crystal
(still disordered)



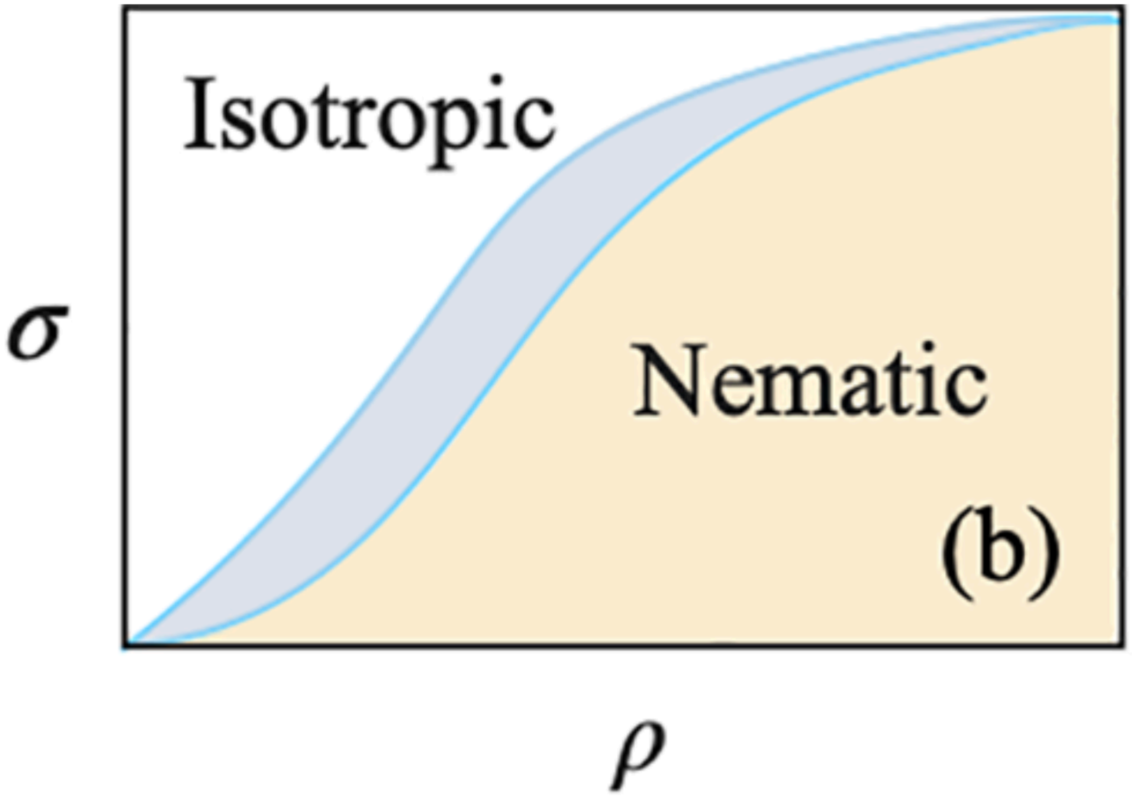
Goal of this talk

- Can one combine **giant density fluctuations** and **hyperuniformity** in the same model?
- Key ingredients: orientational order + absorbing phase transition

Active Nematics



Narayan et al., Science (2007)

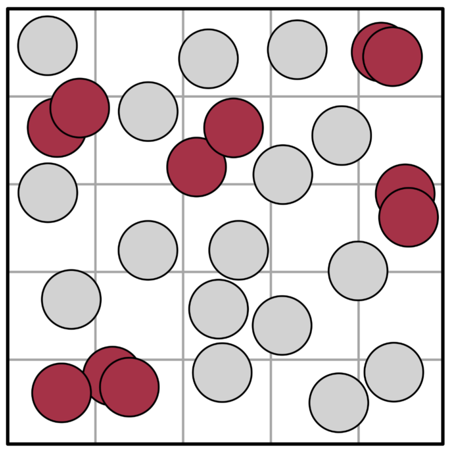


- Discrete time dynamics
- Diffusion along particle's orientation: $\mathbf{r}_i^{t+1} = \mathbf{r}_i^t \pm \delta_0\, \mathbf{e}(\theta_i^t)$
- Nematic alignment: $\theta_i^{t+1} = \frac{1}{2} \arg \left[\sum_{k \in \mathcal{V}_i} e^{i2\theta_k^t} \right] + \psi_i^t \mod \pi$

(noise)

- Noise intensity σ

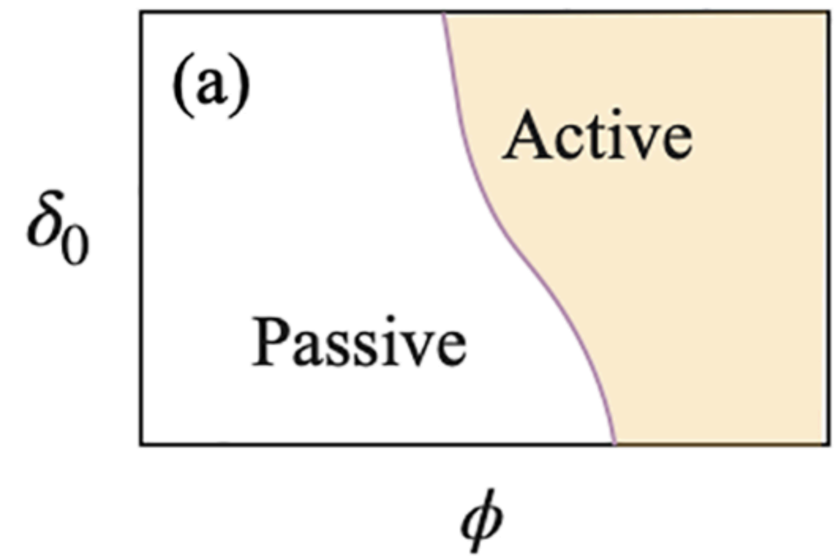
Random Organization Model (ROM)



Tjhung, Berthier, PRL (2015)

Overlapping particles
= active (move)

Isolated particles
= passive (don't move)



- Discrete time dynamics, particles with diameter D

- Only overlapping particles move:

$$\mathbf{r}_i^{t+1} = \mathbf{r}_i^t + \delta_0\,\mathbf{e}(\theta_i^t) \quad \text{with } \theta_i^t \text{ random}$$

- Packing fraction $\phi = N\pi D^2/4L^2$ controls the absorbing phase transition

- No orientational order

Nematic Random Organization Model (NROM)

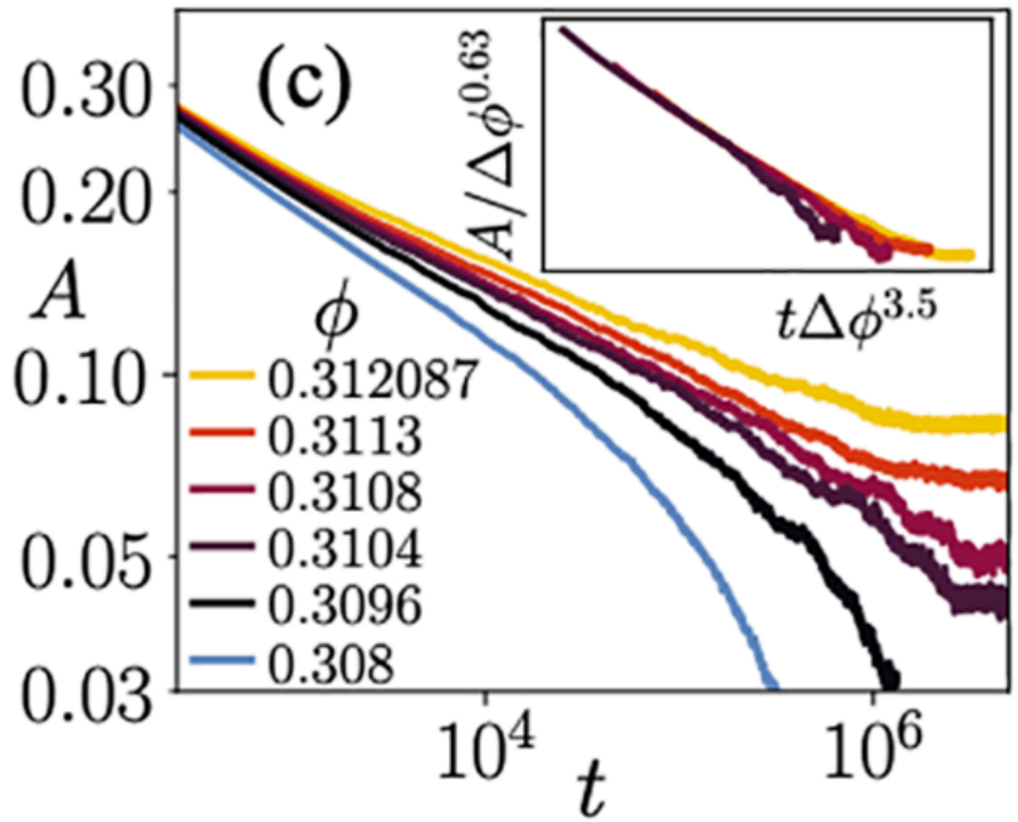
Idea: Combine ingredients from Active Nematics and ROM

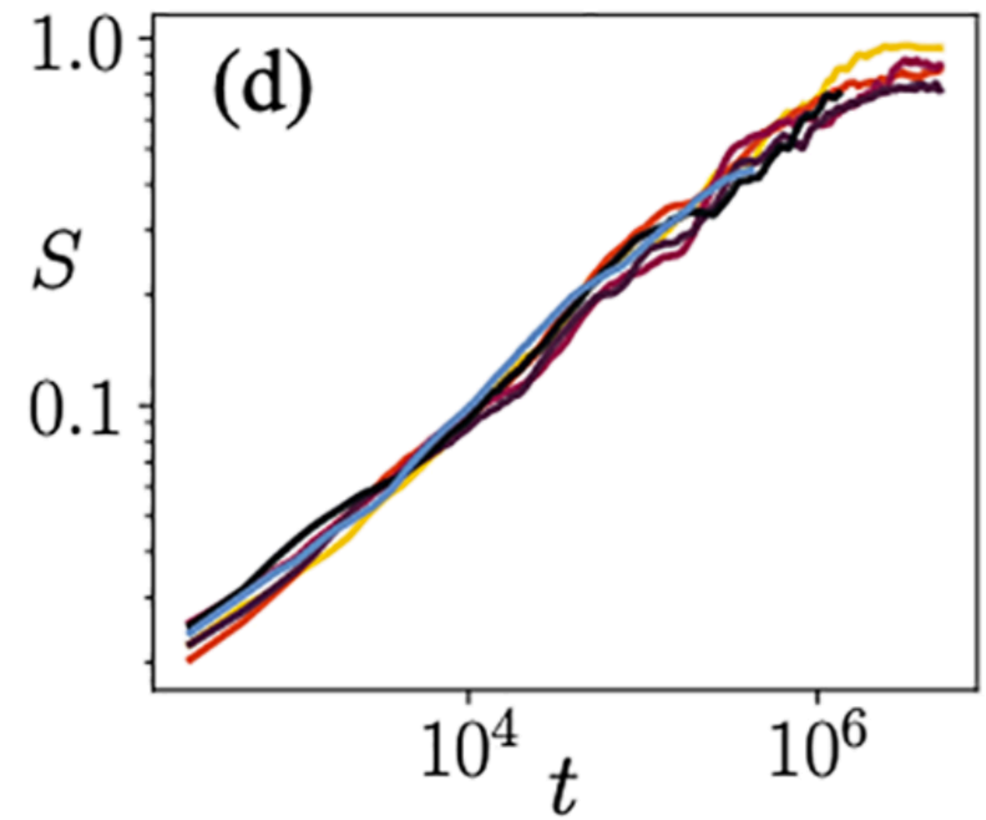
- Particles of diameter D and orientation θ_i
- Nematic alignment with the orientation of neighboring particles (update for all particles)
- Only overlapping particles move: $\mathbf{r}_i^{t+1} = \mathbf{r}_i^t \pm \delta_0\, \mathbf{e}(\theta_i^t)$

Goal: operate in a nematically ordered region of parameter space and look for absorbing phase transition

Activity and nematic order in NROM

Time dependence of mean activity (fraction of active particles) and nematic order for different packing fractions ϕ





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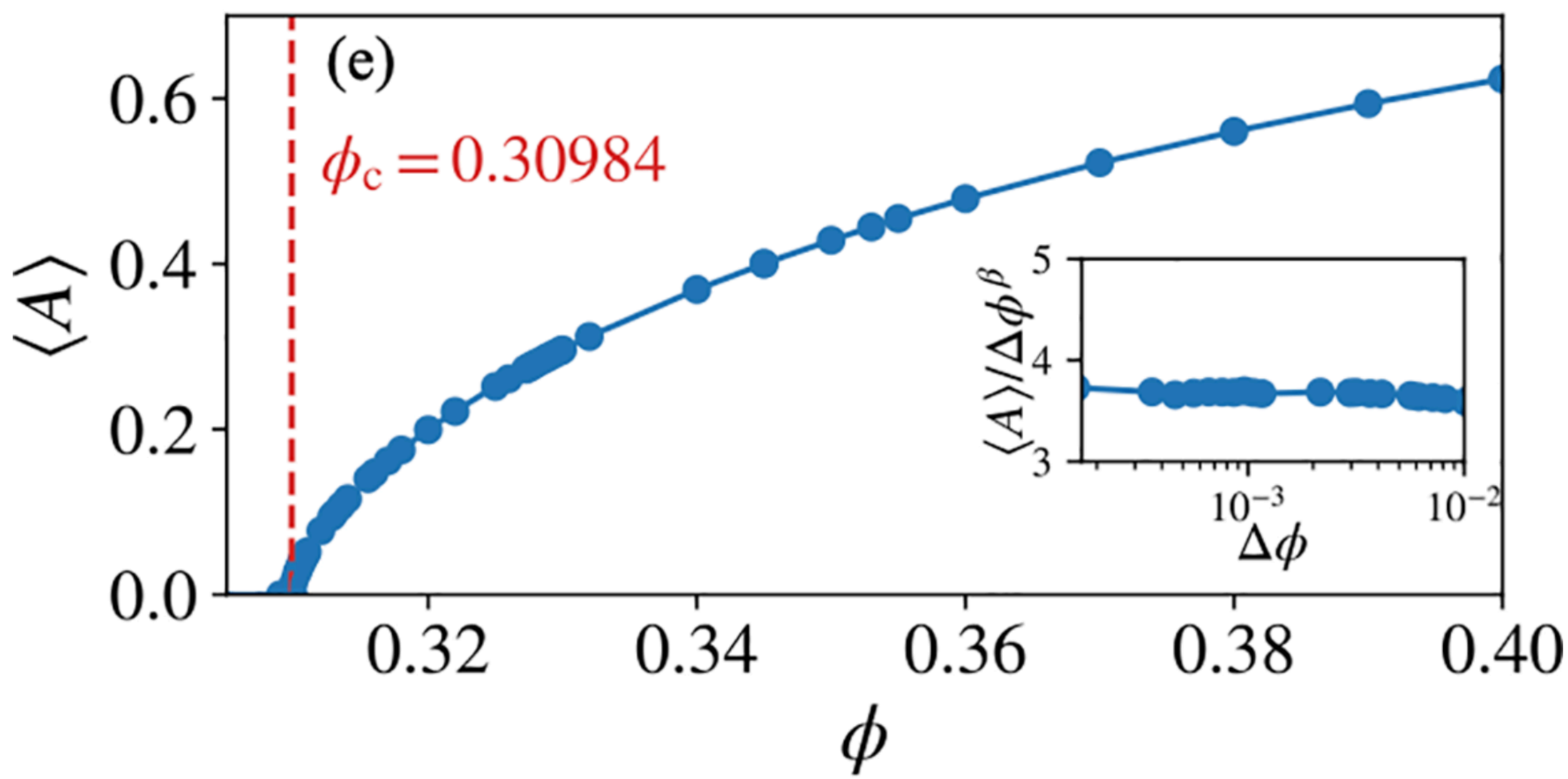
Mean activity in NROM

Stationary mean activity versus packing fraction ϕ

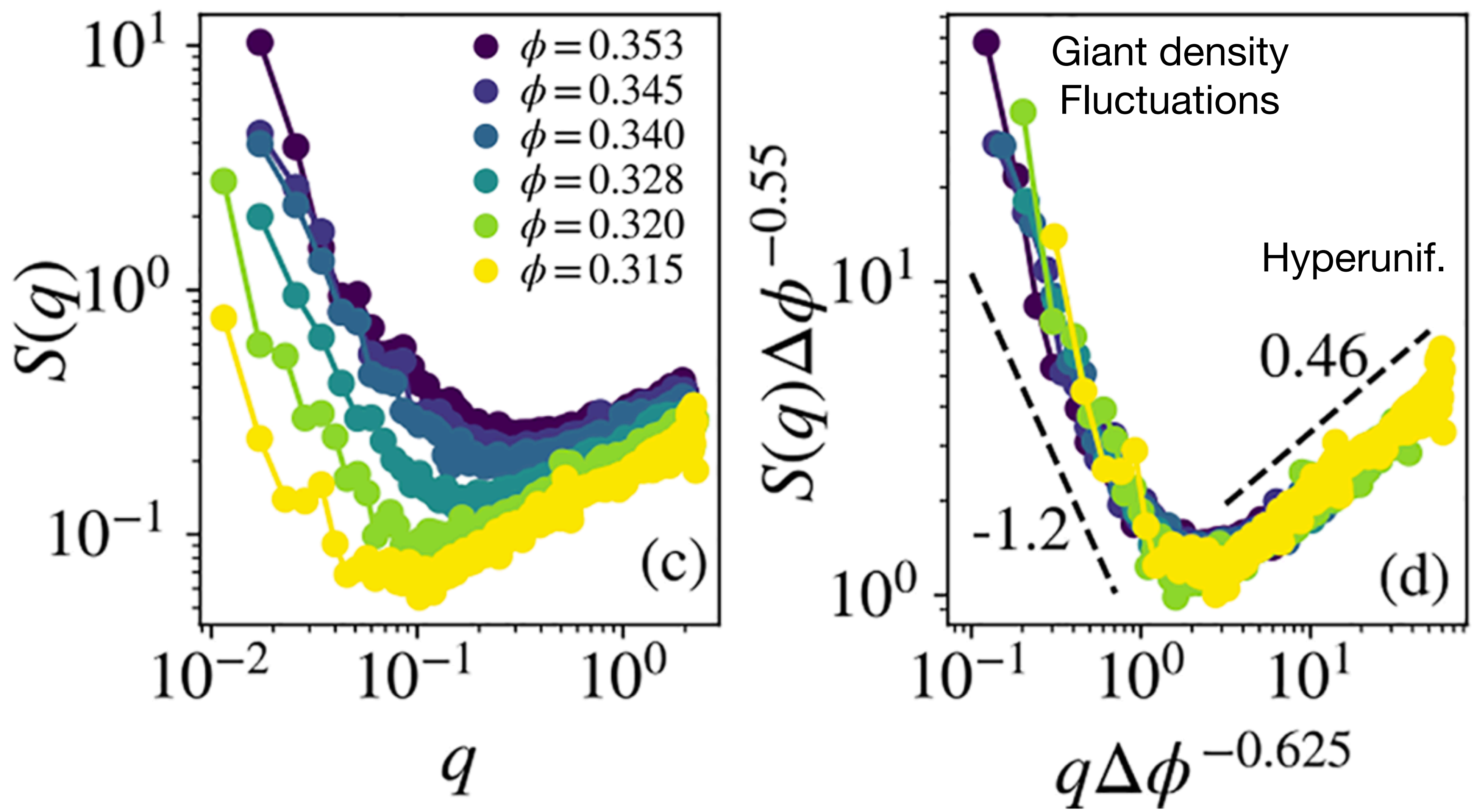


Absorbing phase transition at a critical value ϕ_c

Dal Cengio, Mari, Bertin, PRE (2025)

Density fluctuations: structure factor

$S(\boldsymbol{q}) = N^{-1}\langle \sum_{i,j} e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)} \rangle$ for N particles at position $\mathbf{r}_i$



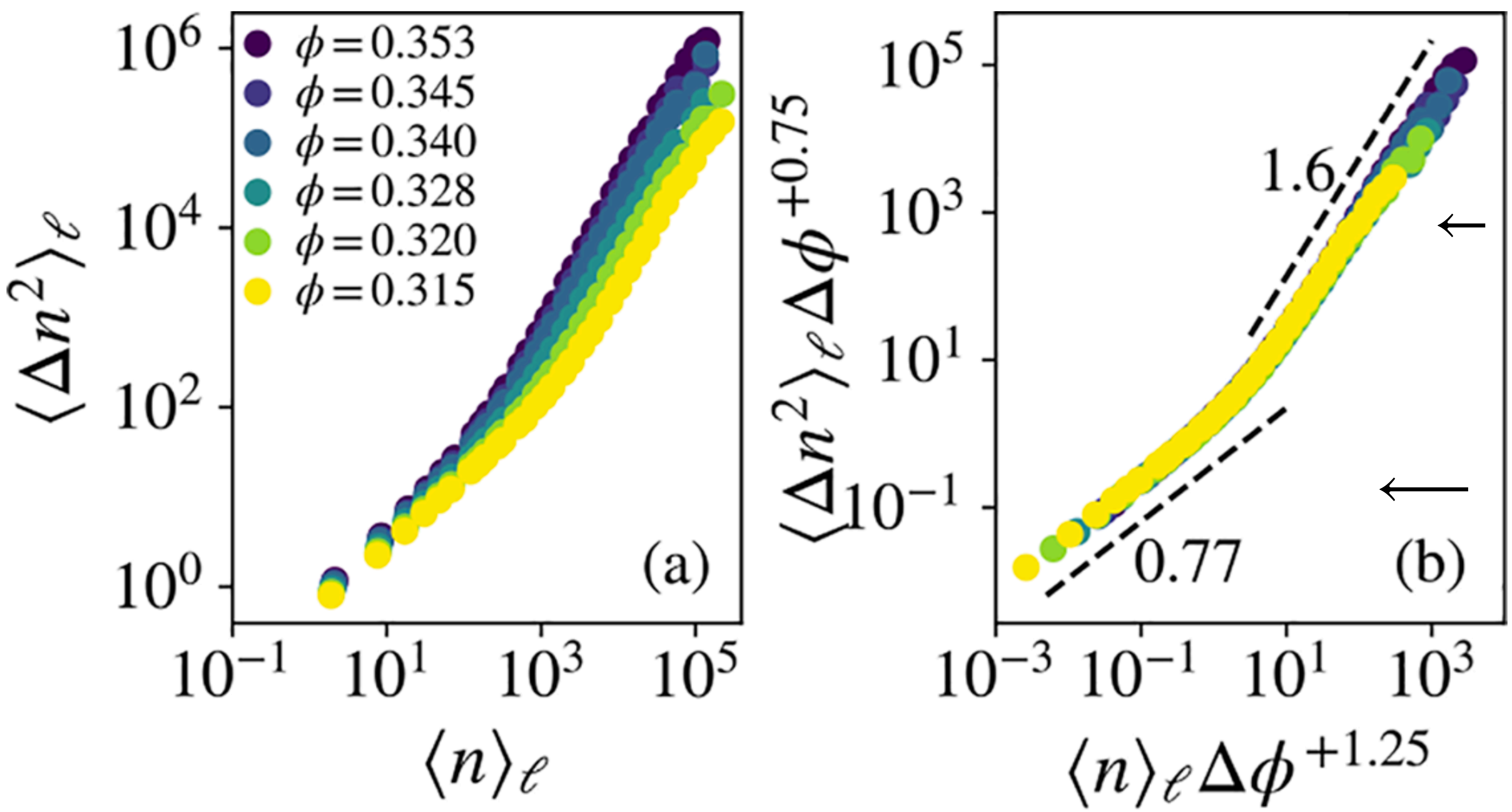
Crossover at $q^* \sim (\phi - \phi_c)^{0.625}$ with $\phi_c \approx 0.310$

Dal Cengio, Mari, Bertin, PRE (2025)

Density fluctuations: box counting method

n particles in a square box of linear size ℓ with $\ell \ll L$

Mean value $\langle n \rangle_\ell$ and variance $\langle \Delta n^2 \rangle_\ell$ with $\Delta n = n - \langle n \rangle_\ell$



← Giant density fluctuations

← Hyperuniformity

Dal Cengio, Mari, Bertin, PRE (2025)

Phenomenological hydrodynamic description

Density field ρ (conserved quantity)

$$\partial_t \rho = \nabla \cdot (\boldsymbol{J} + \sigma_\rho \sqrt{A} \boldsymbol{\eta}_\rho)$$

with flux $\boldsymbol{J} = D_\rho \nabla A + \chi_1 \boldsymbol{Q} \cdot \nabla A + \chi_2 A \nabla \cdot \boldsymbol{Q}$

Activity field A (order parameter absorbing phase transition)

$$\partial_t A = \nabla \cdot \boldsymbol{J} + (\kappa \rho - a) A - \lambda A^2 + \sigma_A \sqrt{A} \eta_A$$

Nematic field $\mathbf{Q}$ (orientational order parameter)

$$\partial_t \boldsymbol{Q} = [\tilde{\mu}(\rho) - \gamma |\boldsymbol{Q}|^2] \boldsymbol{Q} + D_Q \nabla^2 \boldsymbol{Q} + \chi_3 \widehat{\nabla \nabla} A + \sigma_Q \boldsymbol{\eta}_Q$$

Simplified hydrodynamic equations

Linearized equation for $\delta\hat{\rho}(\mathbf{q}, t)$, where $\rho = \rho_0 + \delta\rho$

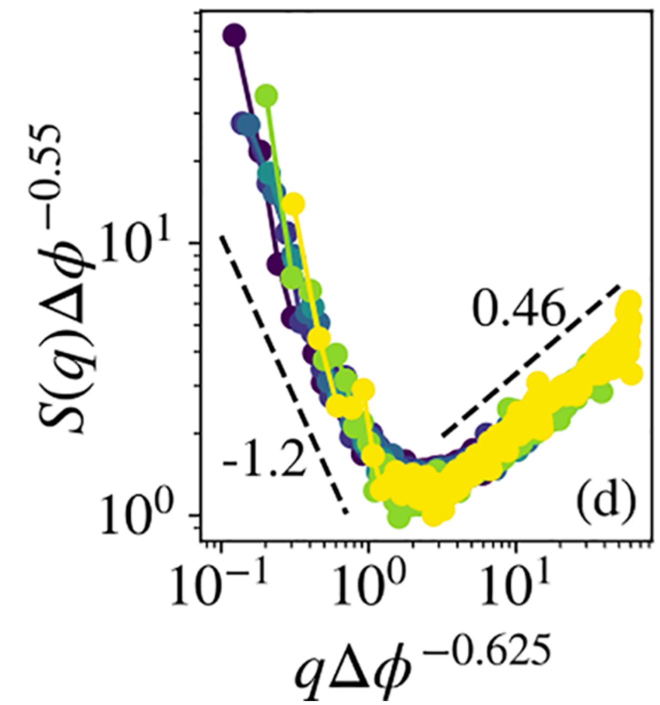
$$\partial_t \delta\hat{\rho} = -Dq^2\delta\hat{\rho} + \frac{\tilde{\sigma}_A q^2}{\sqrt{A_0}}\hat{\eta}_A + \tilde{\sigma}_Q \sin 2\theta A_0 \hat{\eta}_Q^{\perp}$$

$\theta =$ angle $(\mathbf{q}, \mathbf{n_0})$

$\mathbf{n}_0$ nematic director

Activity Noise

Nematic noise



Crossover between two regimes:

$S(q) \sim q^2/A_0$ for $q \gg q^*$: **Hyperuniformity**

$S(q) \sim (1 - \cos 4\theta)A_0^2/q^2$ for $q \ll q^*$: **Giant density fluctuations**

Crossover scale $q^* \sim A_0^{3/4} \sim (\Delta\phi)^{0.75}$

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Summary

- Combine **orientational order** and **absorbing phase transition** in an active matter system
- Crossover between **hyperuniformity** (anomalously low density fluctuations) and **giant density fluctuations** close to the critical point
- Crossover length $\xi^* \sim 1/q^*$ **diverges** at the critical point ϕ_c

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S. Dal Cengio, R. Mari, E. Bertin, Phys. Rev. E **112**, L042101 (2025)