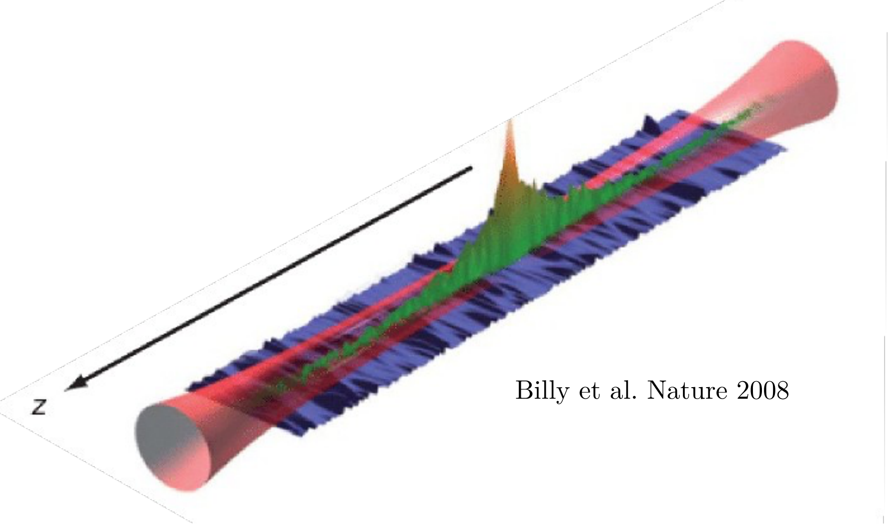
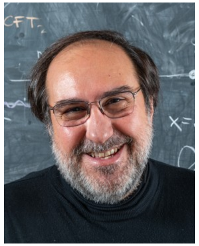


Billy et al. Nature 2008

Laurent Sanchez-Palencia

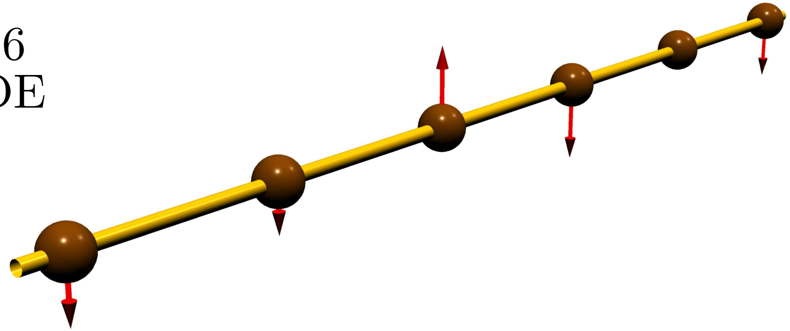


Thierry Giamarchi

Localization Transition for Interacting Quantum Particles in Colored-Noise Disorder

Giacomo Morpurgo

Roscoff, 04/05/2026
5e journées GDR IDE



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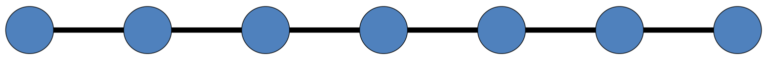
Anderson localization



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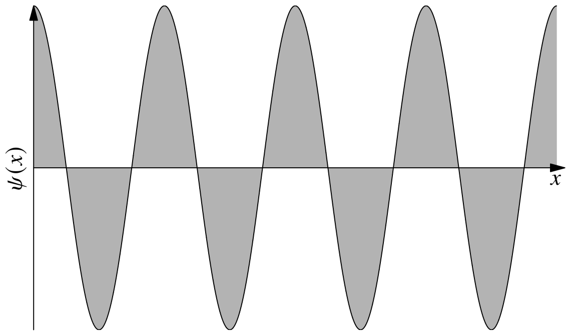

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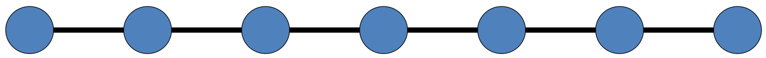

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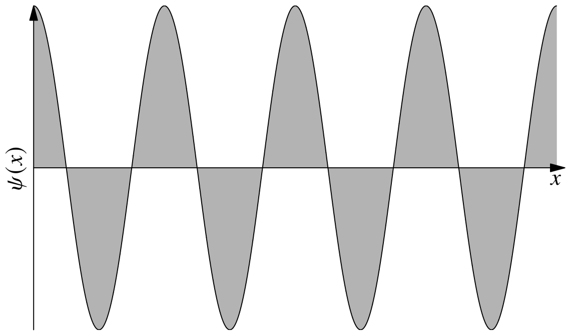
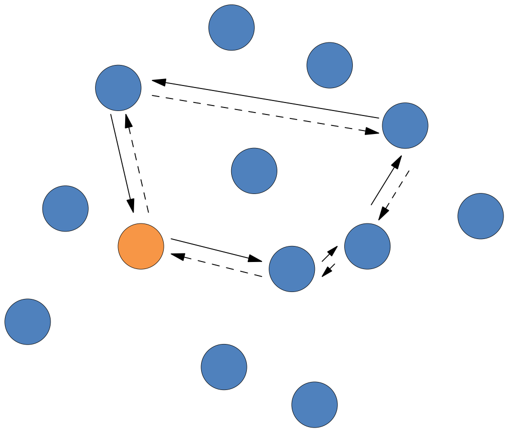
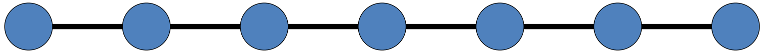


Disordered



- Constructive interference of two opposite paths

Anderson localization



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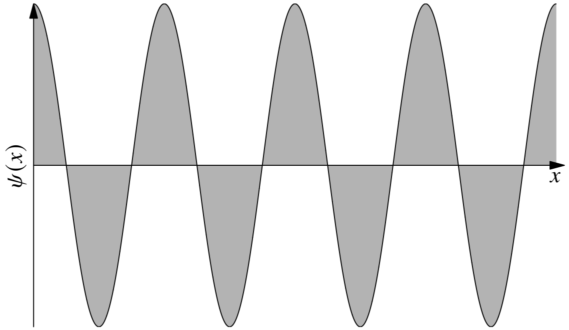

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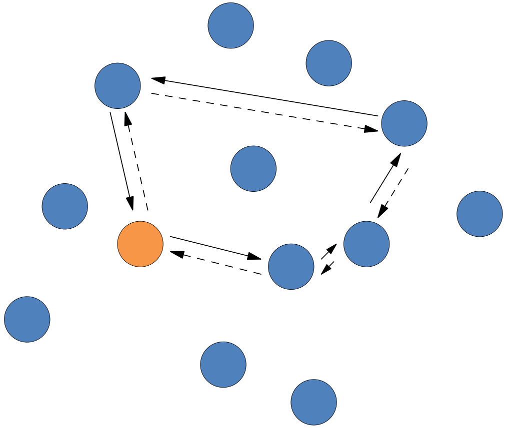

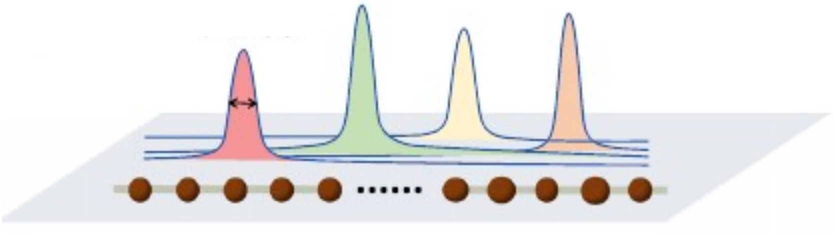

Guo et al.
Nature Communications 2024

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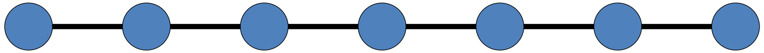

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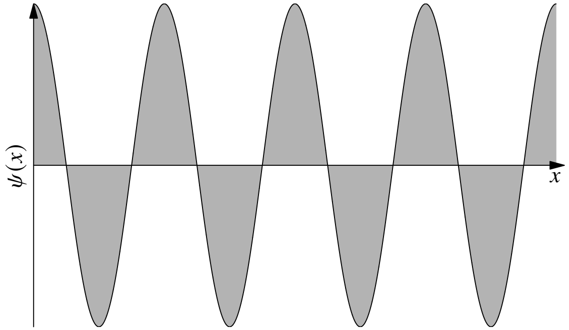

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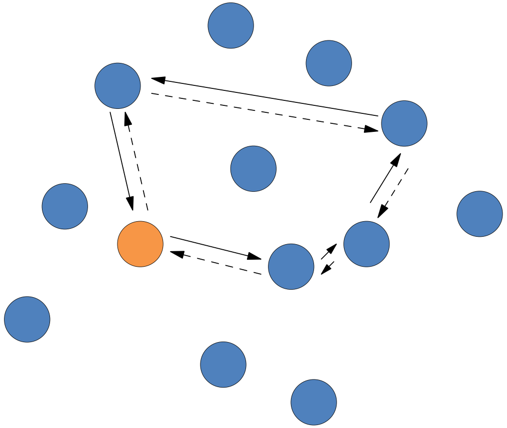

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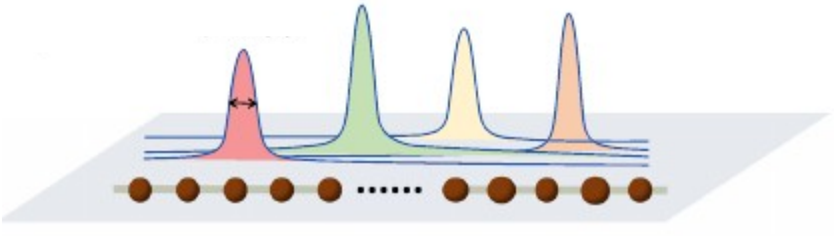
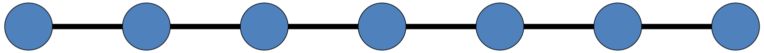


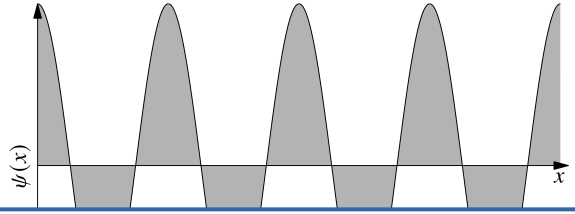
Guo et al.
Nature Communications 2024

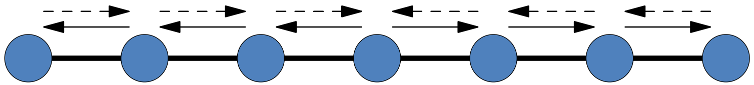
Localization length

$$\psi(x) = \frac{1}{\sqrt{\xi}} e^{-\frac{|x - x_o|}{\xi}}$$

Anderson localization







In 1D, infinitesimal disorder is enough

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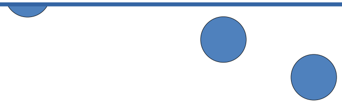

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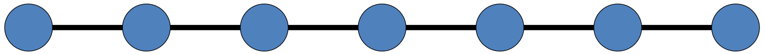

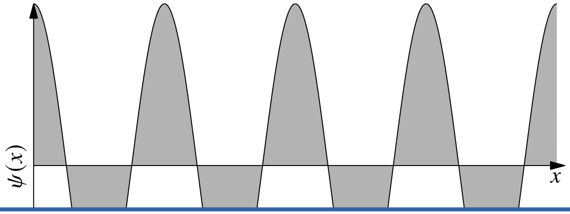

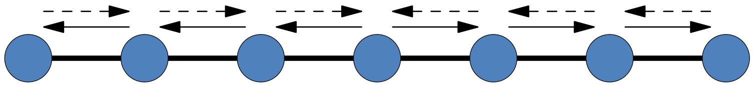

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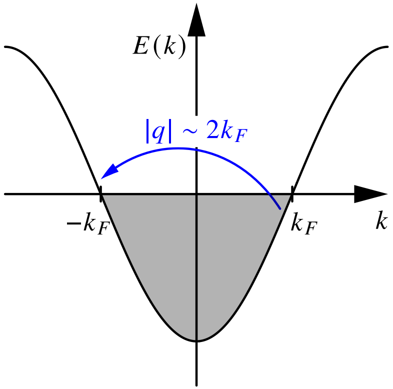

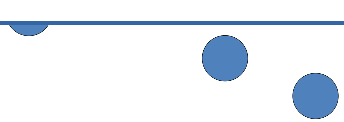

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- Constructive interference
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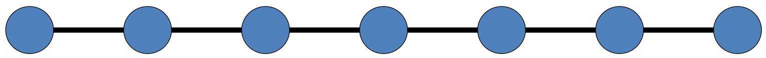
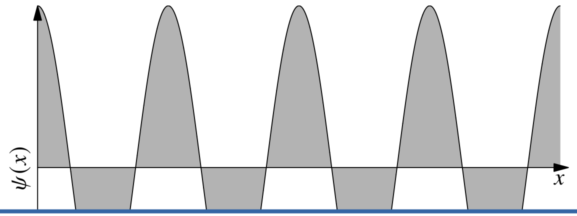
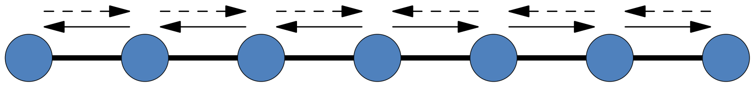
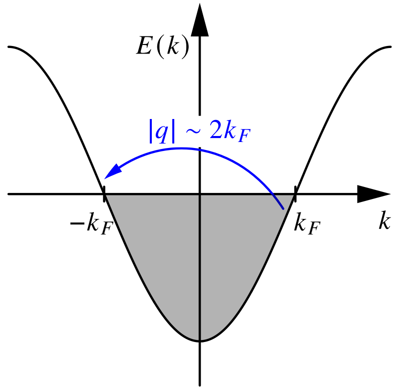


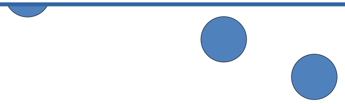




In 1D, infinitesimal disorder is enough

$$\xi \propto \frac{1}{W_0^2}$$





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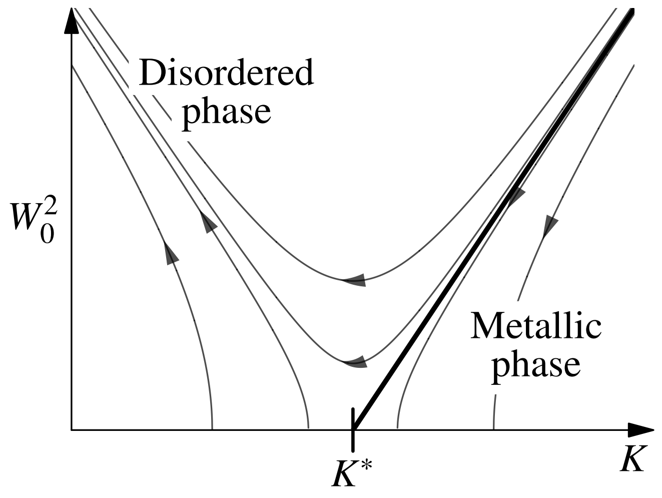
$$\psi(x) = \frac{1}{\sqrt{\xi}} e^{-\frac{|x-x_0|}{\xi}}$$

- Constructive interference of two opposite paths

Adding interactions

Phase diagram



$$K^* = 3/2$$

Adding interactions

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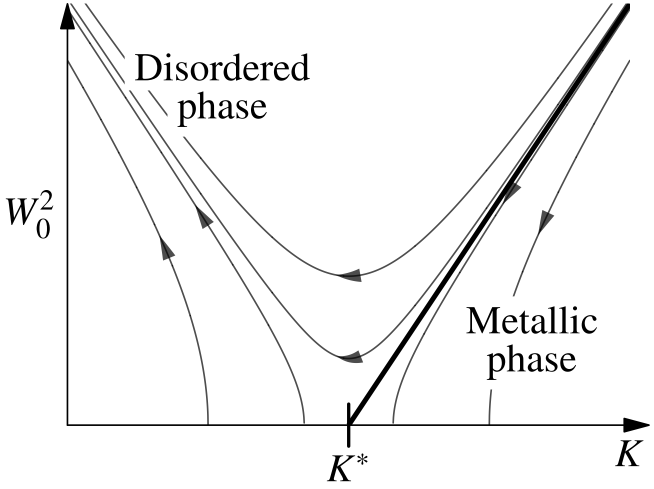


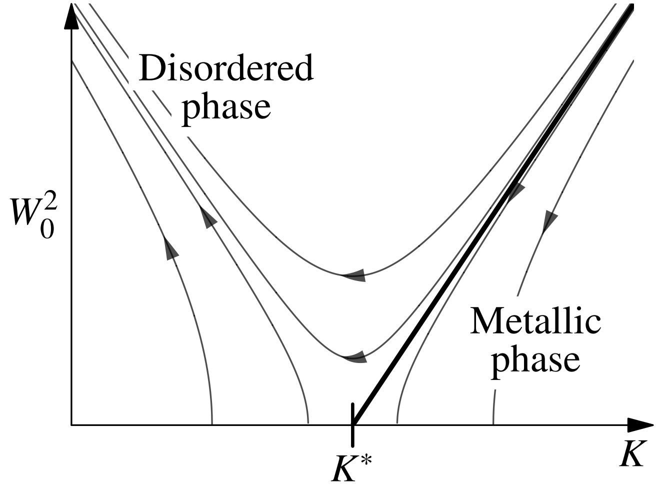
$$K^* = 3/2$$

Localization length

$$\xi \propto \frac{1}{W_0^{2(3-2K)}}$$

Adding interactions

Phase diagram



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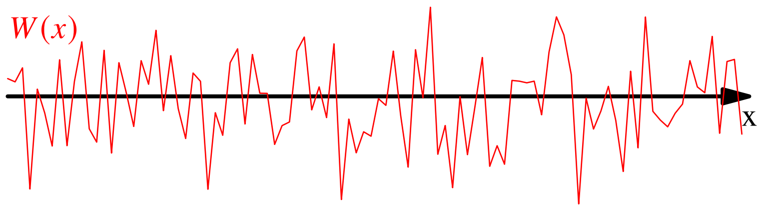
Significant others
-Many body localization
-Bose Glass

Correlations in the disorder

Uncorrelated



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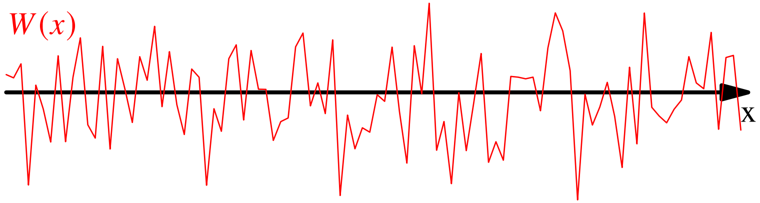
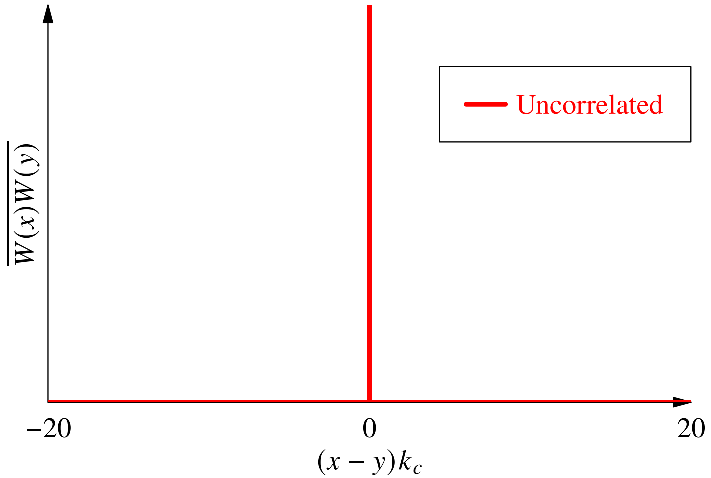
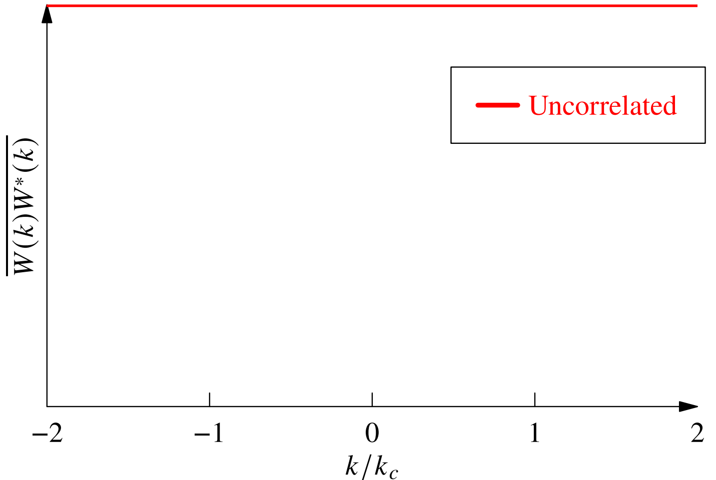


$$\overline{W(k)W^*(k)} = W_0^2\Omega$$





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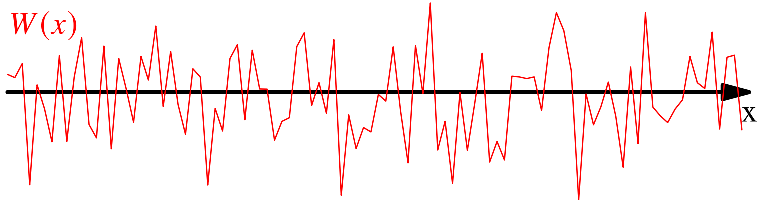

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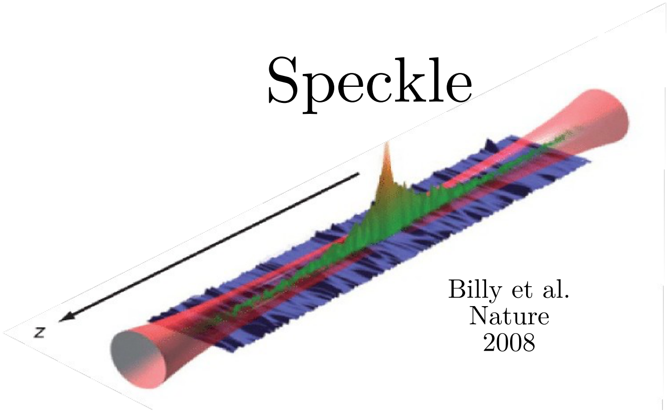
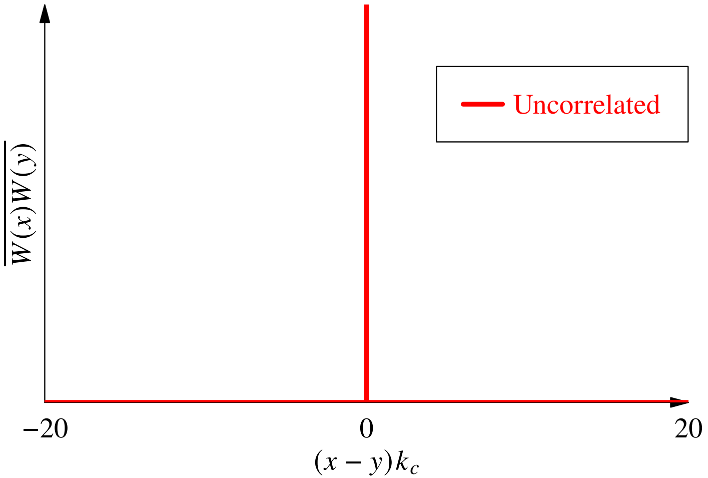
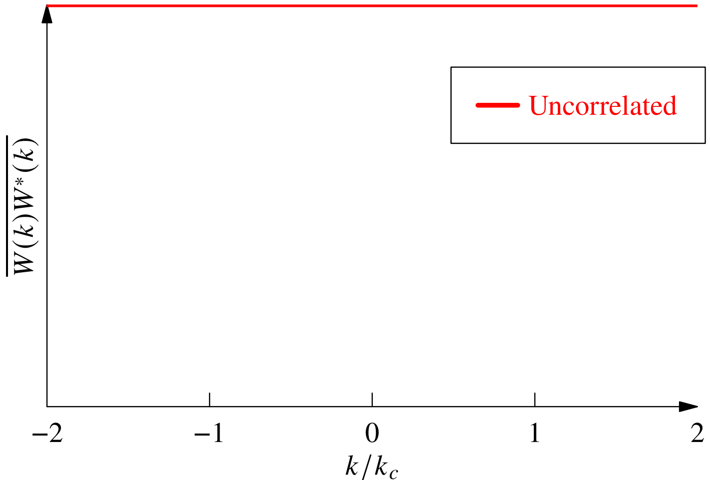
Speckle







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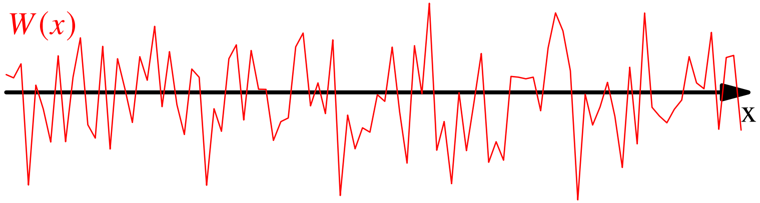

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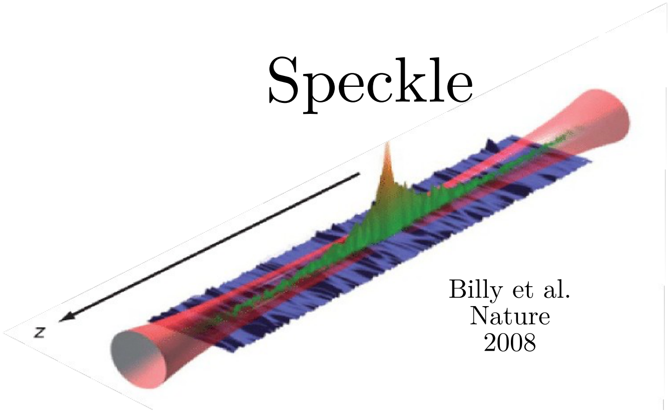
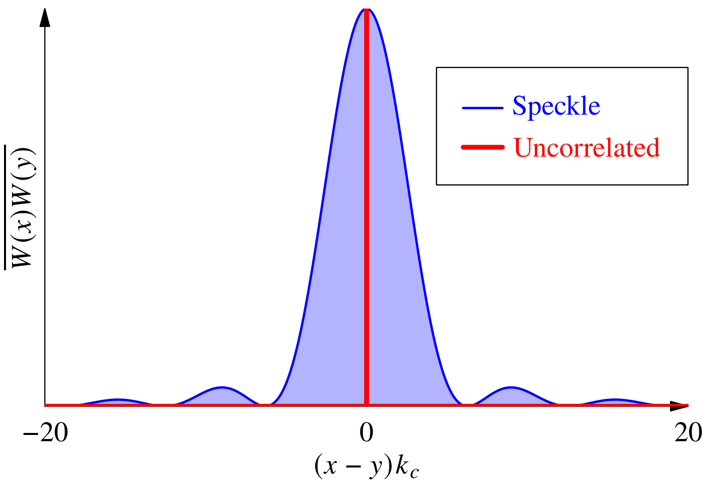


Billy et al.
Nature
2008



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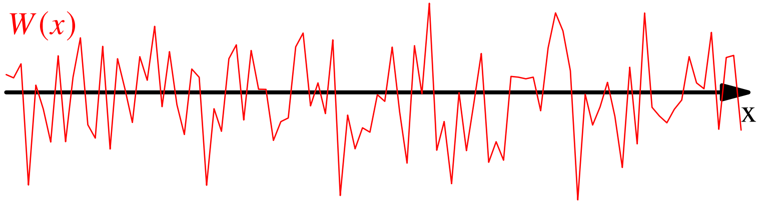

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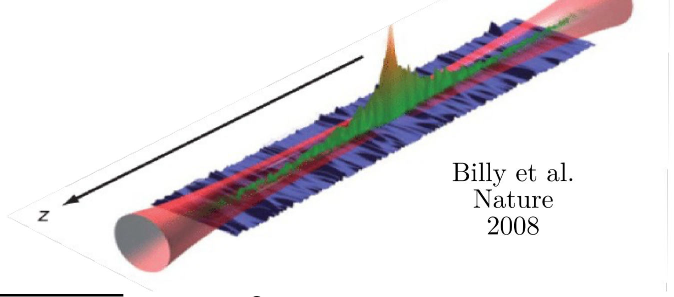
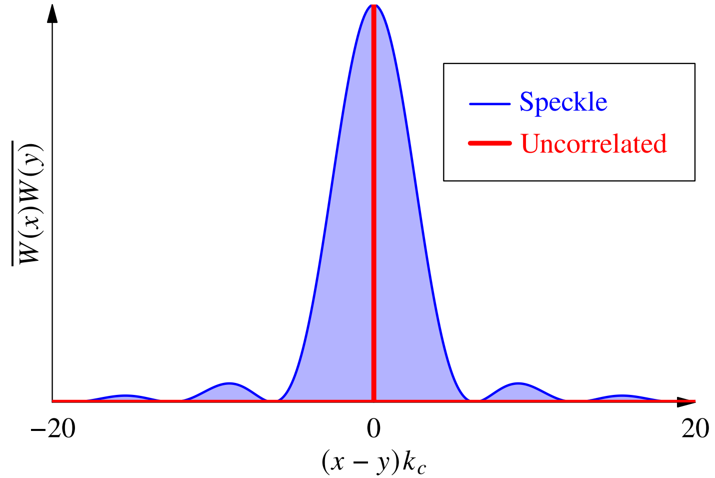
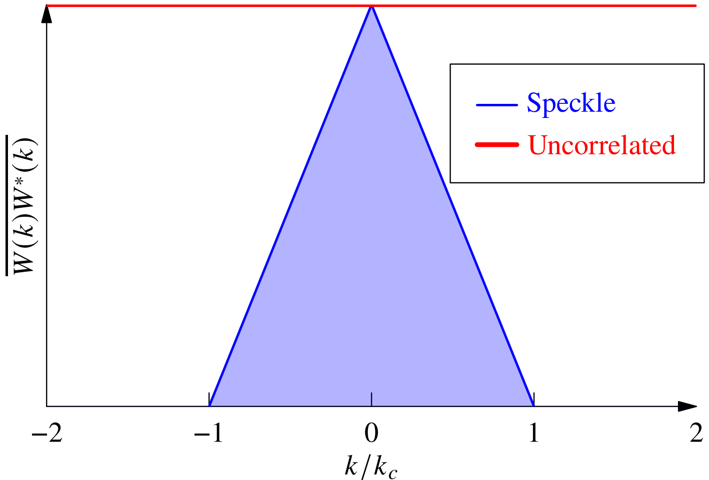
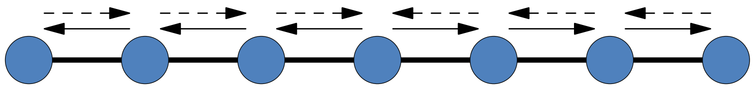


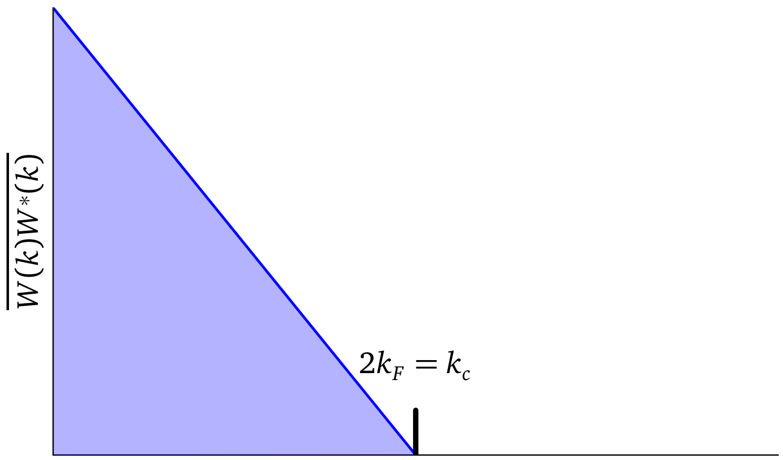
$$\overline{W(k)W^*(k)} = W_0^2\Omega\,(1 - |k|/k_c)\,\theta(k_c - |k|)$$





Our question





Backscattering barely allowed

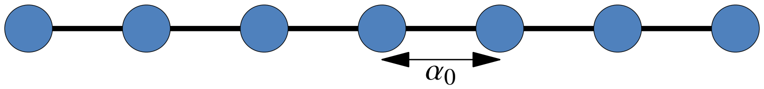
Are there other differences to uncorrelated disorder ?

What is the role of interactions ?

2 angles of attack

Bosonized Hamiltonian

Low-energy excitations are collective modes (in 1D)



$$H = \overbrace{\frac{1}{2\pi}\int dx\left(uK\left(\nabla\theta(x)\right)^2 + \frac{u}{K}\left(\nabla\phi(x)\right)^2\right)}^{H_{\text{kin}}+H_{\text{int}}} + \overbrace{\frac{1}{2\pi\alpha_0}\int dx W(x)\cos\left(2k_F x - 2\phi(x)\right)}^{H_{\text{dis}}}$$

Bosonic fields

$\phi(x)$ describes the density

$\theta(x)$ describes the phase

Interactions

K

Non-quadratic term became quadratic

Disorder

Strength given by $W(x)$

Non-quadratic term

RG equations

Valid only for small disorder strengths W_0

Integrating over the degrees of freedom $\alpha = \alpha_0 e^l$

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$$\frac{\partial K}{\partial l} \propto -\frac{K^2 W_0^2}{4} \sum_k (1 - |k|/k_c)\, \theta(k_c - |k|) \left[J_0\big((k + 2k_F)\alpha\big) + J_0\big((k - 2k_F)\alpha\big) \right]$$

$$\frac{\partial W_0^2}{\partial l} = (4 - 2K) W_0^2$$

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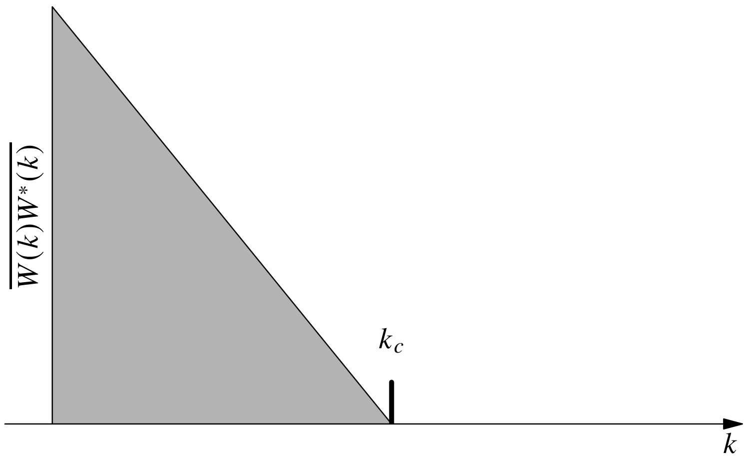

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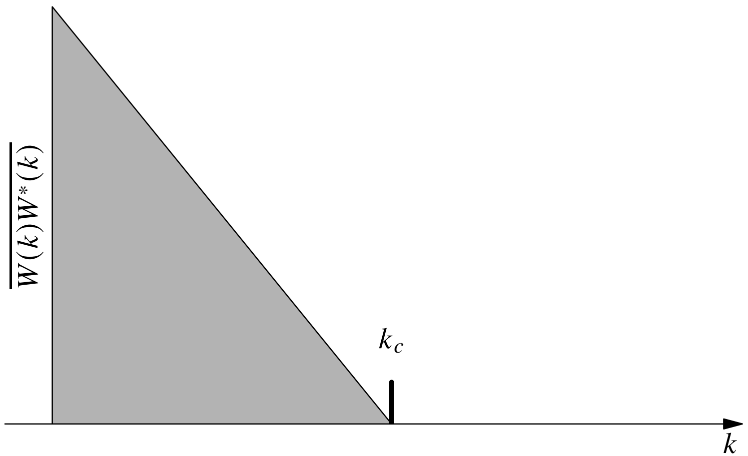

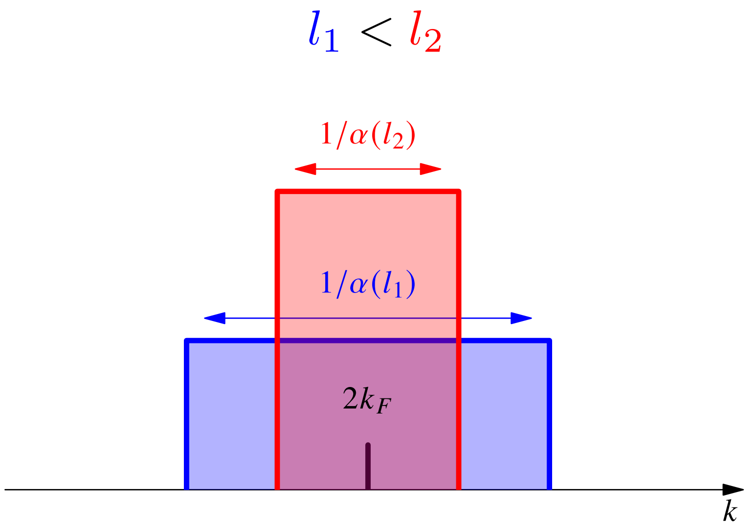

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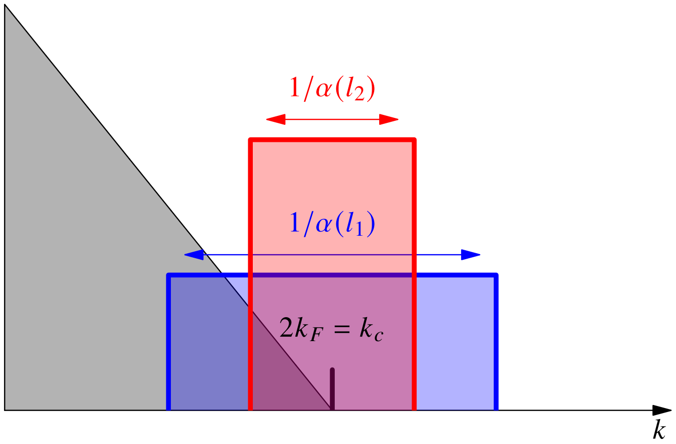

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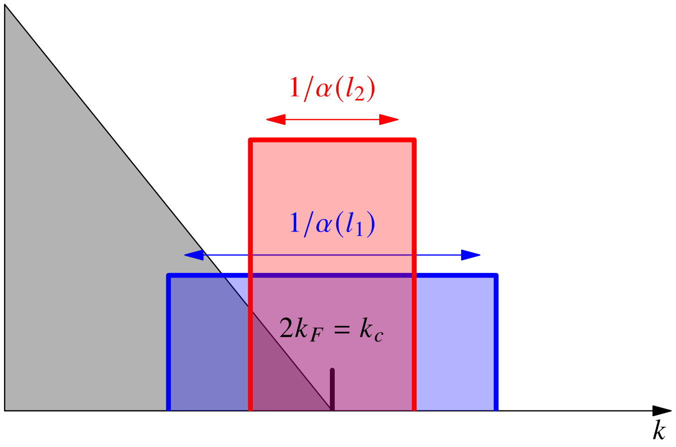

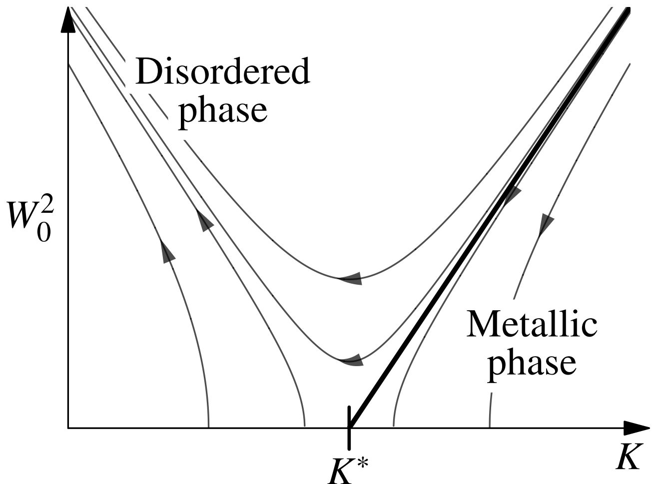

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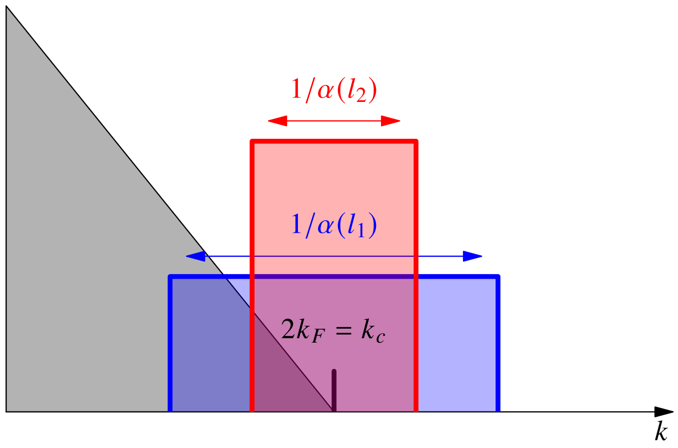
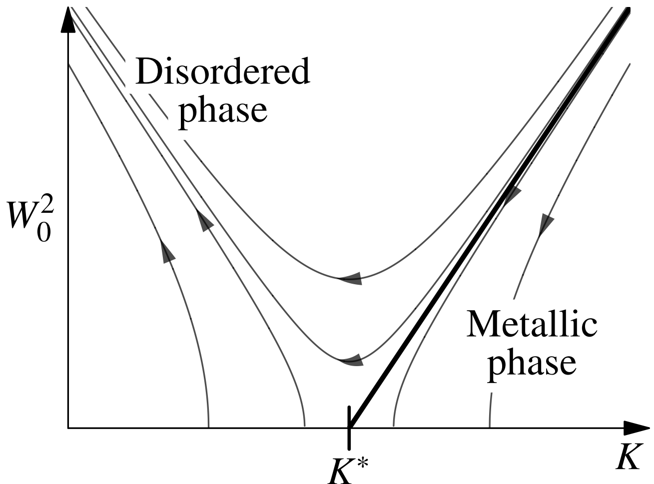


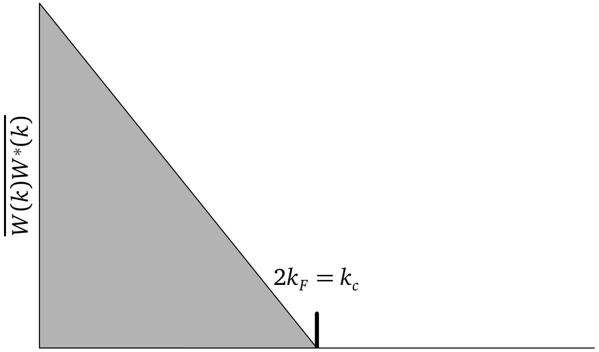


~~$K^* = 3/2$~~

$K^* = 1$

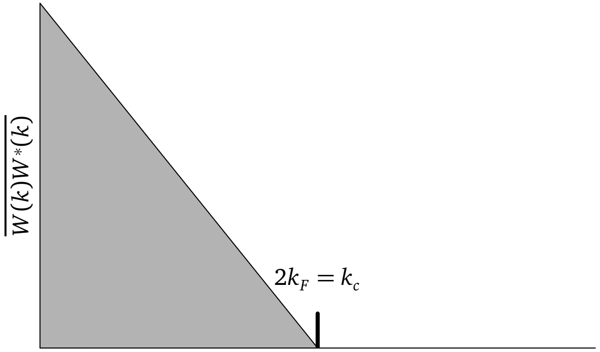
Shift of transition

K^* dependence of the correlations-shape



$$\overline{W(k)^*W(k)} = W_0^2 \left(1 - \frac{|k|}{k_c}\right)^{\nu} \theta(k_c - |k|)$$

K^* dependence of the correlations-shape



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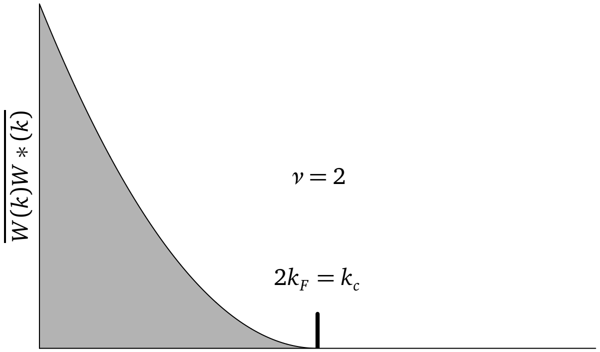
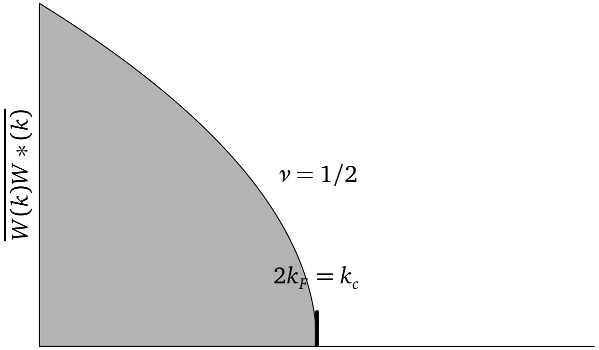
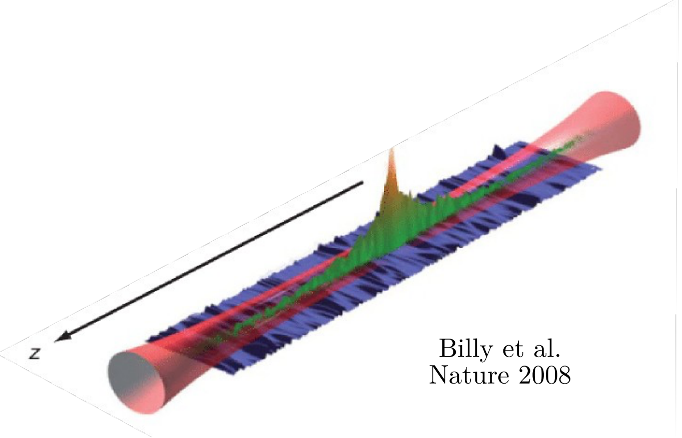




$$K^* = \frac{3 - \nu}{2}$$

Localization length scaling



Billy et al.
Nature 2008

$$\psi(x) = \frac{1}{\sqrt{\xi}} e^{-\frac{|x - x_o|}{\xi}}$$

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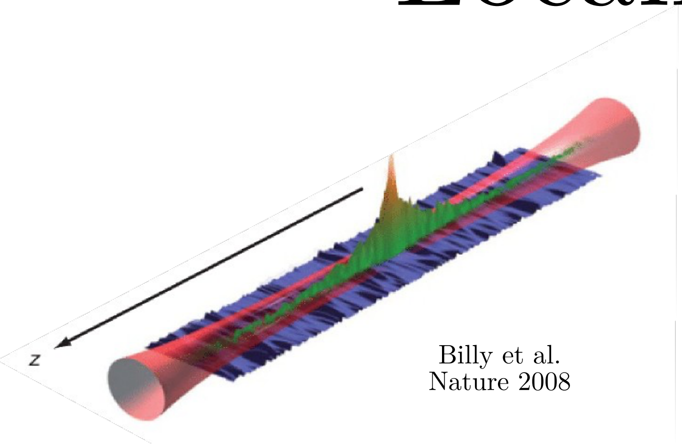
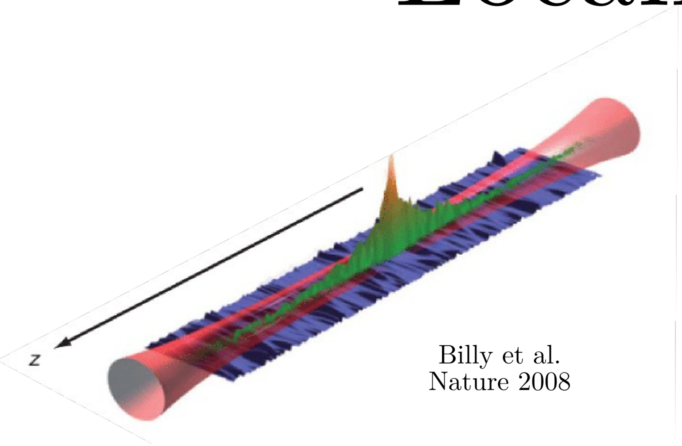


Billy et al.
Nature 2008

$$\psi(x) = \frac{1}{\sqrt{\xi}} e^{-\frac{|x-x_o|}{\xi}}$$

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_i W_i \, c_i^\dagger c_i$$

Localization length scaling



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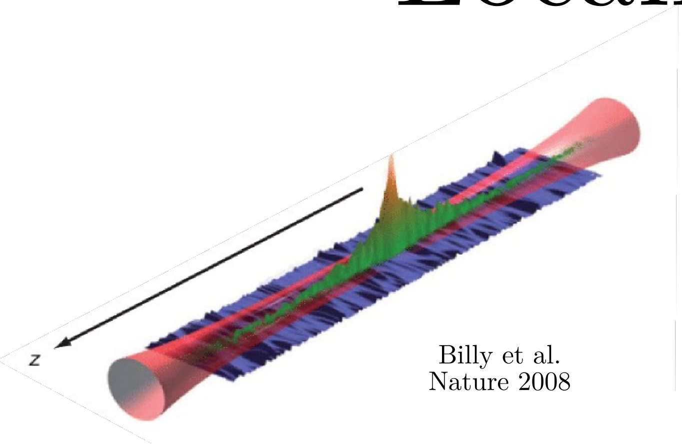
Look at the scaling of $\xi(W_0^2)$

Localization length scaling



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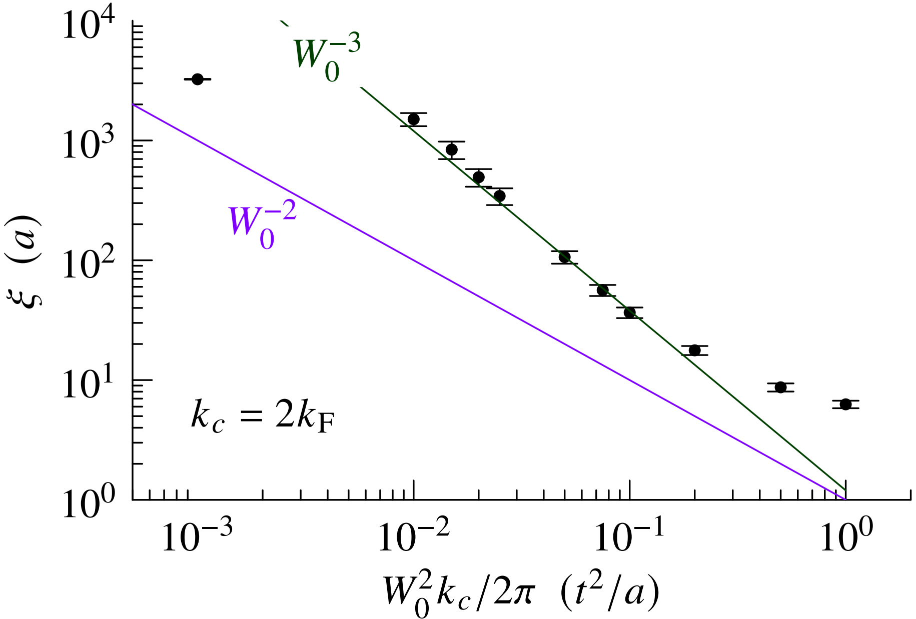

Conclusions and outlook

- Vanishing backward processes lead to deviations from properties of uncorrelated disorder

- Transition from a metallic phase to a disorder dominated phase is shifted to the non-interacting point

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- Understand different localization length scaling analytically

- Look at other observable quantities

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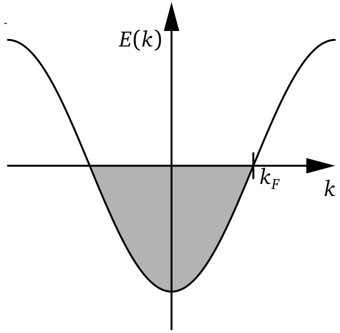
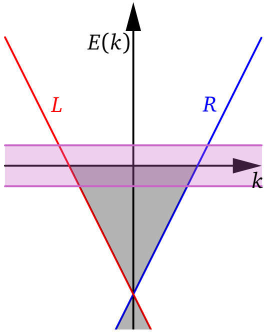
Thank you for your attention !

Hamiltonian





$$H = H_{\text{kin}} + H_{\text{int}} + H_{\text{dis}}$$

$$H_{\text{kin}} = \sum_r \sum_k v_{\text{F}}(\varepsilon_r k - k_{\text{F}}) c^\dagger_{r,k} c_{r,k}$$

Linear kinetic part

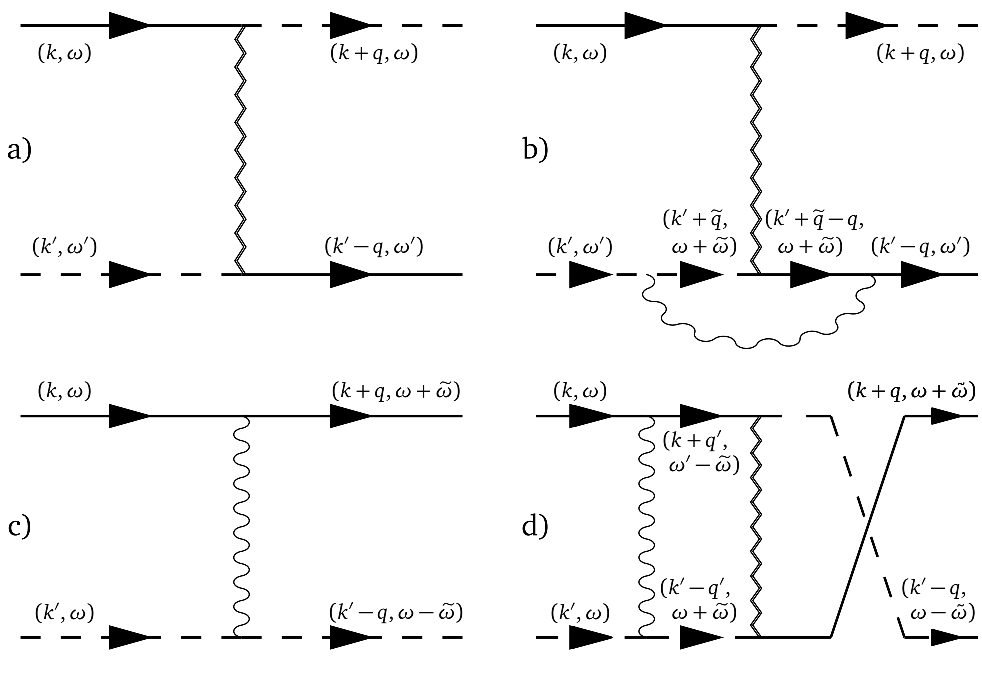
$$H_{\text{int}} = \frac{g}{2\Omega} \sum_r \sum_{k,k',q} c^\dagger_{r,k+q} c^\dagger_{-r,k'-q} c_{-r,k'} c_{r,k}$$

Interactions between left and right movers

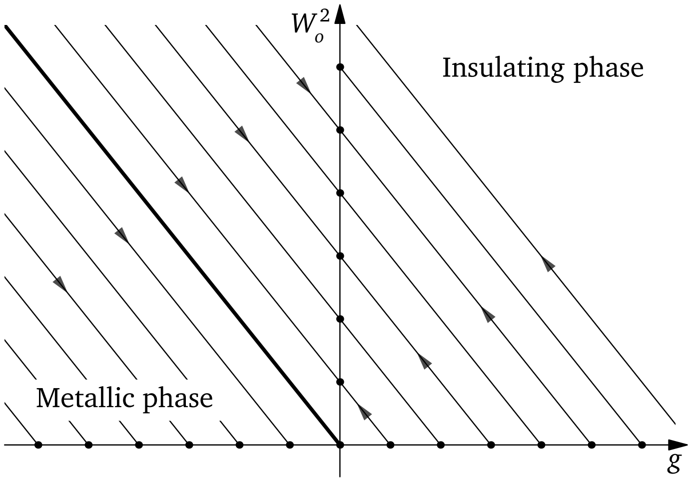
$$H_{\text{dis}} = \frac{1}{\Omega} \sum_{k,q\sim 0} \left[W_{q-2k_{\text{F}}} c^\dagger_{R,k+q} c_{L,k} + W_{q+2k_{\text{F}}} c^\dagger_{L,k+q} c_{R,k} \right]$$

Disorder processes change left into right movers

RG perturbative procedure



$$\begin{cases} \frac{\partial g}{\partial l} & \propto -gW_0^2 \\ \frac{\partial W_0^2}{\partial l} & \propto gW_0^2 \end{cases}$$

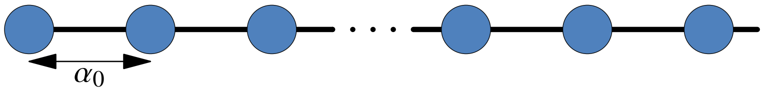


Renormalization Group (What is it?)

Idea :

- Complicated theory to investigate

- Interested only in macroscopic properties



- Integrate over some of the degrees of freedom

- Get a “new” theory (hopefully simpler)

- Repeat until you get a theory which can be treated

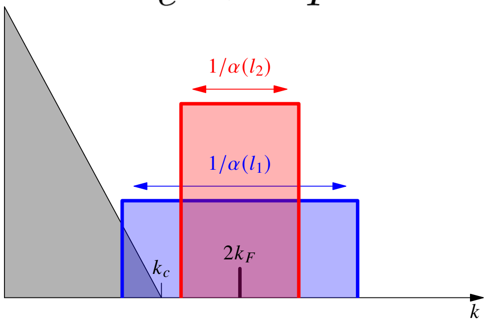
$$H(K, W_0)$$

$$H^{\text{eff}}(K^{\text{eff}}, W_0^{\text{eff}}) = H(K^{\text{eff}}, W_0^{\text{eff}})$$

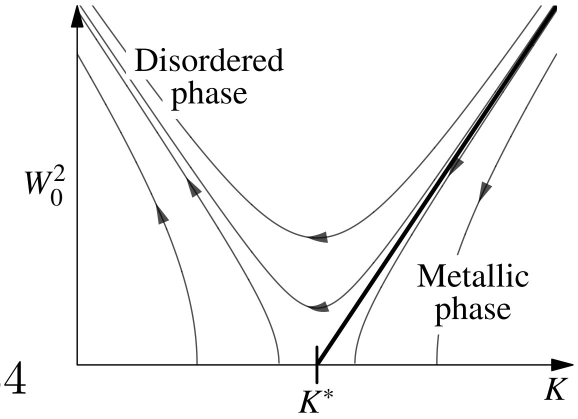
Looking for RG equations for the parameters of the theory K, W_0

RG equations : Flow of 3 cases

$k_c < 2k_F$



"Non-disordered"



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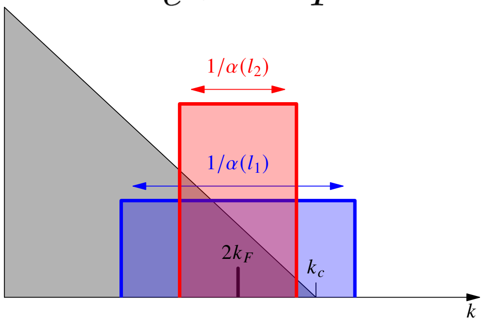
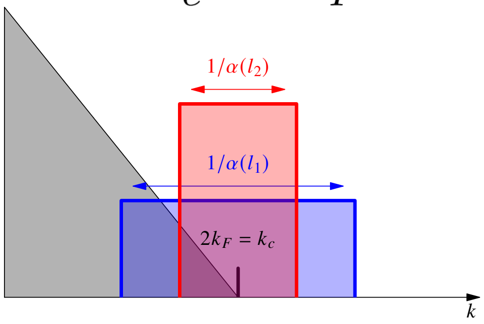


$$\frac{\partial K}{\partial l} = -K^2 C e^{(3-2K)l}$$

Same behaviour as uncorrelated disorder

$$K^* = 3/2$$

$k_c = 2k_F$



$$\frac{\partial K}{\partial l} = -K^2 C e^{(2-2K)l}$$

Different critical interaction

$$K^* = 1$$

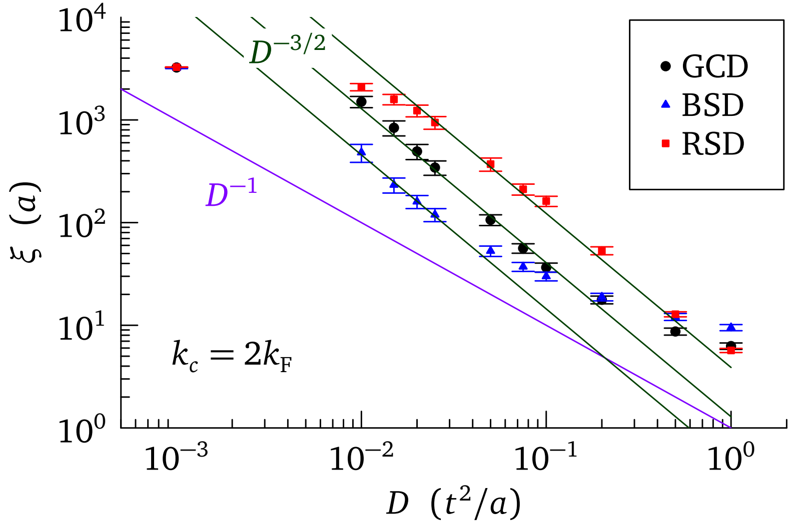
Transition point shifted to non-interacting system

Localization length scaling

Compute ξ with IPR

$$\mathrm{IPR}(E) = \int dx |\psi(x,E)|^4$$

3 “correlated speckle-like disorders”



Size-dependence